A THEORETICAL MODEL OF ULTRASONIC EXAMINATION OF SMOOTH FLAT CRACKS

R.K. Chapman and J.M. Coffey

CEGB NDT Applications Centre
Timpson Road
Manchester, M23 9LL, England

INTRODUCTION

As the Introductory Paper to this Conference explains,[1] the CEGB and other high technology organisations are very interested in quantitative NDE for the guidance it gives in the design of inspections and for the support it offers in Safety Submissions to the Regulatory Authorities. An important part of this work is the theoretical modelling and prediction of defect detectability and signal behaviour. This present paper complements the Introductory Paper by describing the technical content of a model we have developed at the CEGB NDT Applications Centre.

Our aim has been to devise a practical, yet accurate and reliable model for the overall inspection process which can be readily adapted to different inspection geometries and conditions, and which does not involve an inordinate amount of computing time. Our model includes the following aspects of the inspection: a model of the probe and the beam it generates, the geometry of the surfaces over which the probe is scanned, the scattering of the probe beam by arbitrarily oriented smooth planar defects and by geometrical features of the test component, and the detection of the scattered signal by the receiving probe. These aspects are outlined below; a rather longer account has been published by the authors,[2] while full details of most aspects with numerical examples are given in CEGB Reports.[3-5] Our aim of developing a balanced, integrated model in which all the salient features of the inspection are represented has strong parallels with the work of Thompson and colleagues reported at this conference,[6] and contrasts with the type of study widely represented in the literature which concentrates on giving a detailed description or analysis of a narrow aspect of the ultrasonic inspection process.

Clearly any reliable overall model must include an accurate description of the ultrasonic waves and the defect. On this matter there are a few rather philosophical points which we would like to discuss before turning to the technical features in the next Section. We are concerned with the accuracy, realism and hence the validity of the model's predictions. We can conveniently think of there being two aspects to this question: firstly, how well does the idealised model defect represent natural metallurgical defects and secondly, how accurately can we predict the scattered signals from such an idealised model defect? These two aspects are best investigated separately since if inaccuracies do occur in applying the model to real defects, this facilitates recognition of where the inaccuracies arise.

The first aspect of model validity can be investigated by comparing the idealised crack with metallurgical descriptions of real defects of the kind likely to occur. There is an outstanding need for metallurgical evidence on the nature, morphology, orientation and occurrence of the various defect types to be compiled in such a way that the range of applicability of theoretical models can be assessed. Our model initially takes the idealised crack to be a single smooth planar cut with non-contacting, stress-free faces in a homogeneous, isotropic elastic solid obeying the laws of linear elastodynamics. Fatigue cracks, lack-of-fusion defects (in the absence of slag entrapment), and probably weld solidification cracks are expected to be closely modelled in this way. The effects of crack roughness are later included in the model description using experimentally determined corrections.

Related to this point is the question of the validity of artificially introduced reflectors in test blocks. We recognise that rather contrived methods may be necessary to introduce defects into a test specimen, but caution must be exercised in drawing conclusions. For instance a saw cut or flat-bottomed hole will represent a smooth flat crack reasonably well for signals scattered near to the specular direction, but may bear little similarity to a crack viewed well away from the specular direction since the edge structures of slot, flat-bottomed hole and crack are very different. Moreover, it is the weaker signals observed in non-specular directions which it is important to model correctly in assessing an inspection procedure, since these are more likely to border the detection threshold and so be sensitive to inaccuracies.

The second aspect of model validity concerns how accurately it describes the signals from the idealised model defects. Exact analytical or numerical solutions to the elastic wave equation and boundary conditions are known for only the few simplest cases, and the models must therefore rely on approximate theories of elastic wave scattering, such as those described in the next Section. In general, it is not possible to make rigorous quantitative statements about the errors incurred in using these approximate theories, and it is therefore important to compare the approximate solutions with exact solutions for those cases where such exact solutions are available. Consequently, this aspect of assessing a model's validity is most appropriately performed mathematically, since direct assessment by comparison with experiment is complicated by the problems mentioned above of ensuring that the experimental circumstances are equivalent to the theoretical premises.

BRIEF ACCOUNT OF THE MODEL

In this short paper we only outline the main features of our model of ultrasonic inspection. Fuller details are given elsewhere.[2-5] The model combines approximate descriptions of the defect, the defect-sound interaction, and the transmission and reception of the sound by the probes, all in a framework of the component geometry.

Scattering from Cracks

We currently use two approximate theories to describe defect-sound interaction : elastodynamic Kirchhoff diffraction theory for defects viewed in specular or near-specular directions, and an elastodynamic version of Keller's Geometrical Theory of Diffraction[7] for defects viewed in more off-specular directions.

The first step in the elastodynamic Kirchhoff theory is the Elastic Green's Theorem, which is an exact integral representation of the scattered field at a receiving point $\underline{r}'$ in terms of the dynamic crack opening displacement (COD), $[\underline{u}(\underline{r})]$, over the surface A of the crack. For a stress-free crack this has the form (Ref. 8, pp. 33-38):

$$u_p(\underline{r}') = \int_A [u_i(\underline{r})]\, \Sigma_{ij}^{(p)}(\underline{r},\underline{r}')\, n_j(\underline{r})\, d^2\underline{r} \ , \tag{1}$$

where $\Sigma_{ij}^{(p)}(\underline{r},\underline{r}')$ is the Green function describing the stress field at $\underline{r}$ due to a point force at $\underline{r}'$ (Ref. 8, p25). We will use this equation to introduce two of the plausible approximations in our model. The first concerns the extension of equation (1) which applies to a probe rather than to a point receiver, and for which the integrating effect of the finite crystal area must be taken into account. By reciprocity arguments similar to those expressed by Kino[9], which use the property that the probe's behaviour on reception is closely related to its behaviour on transmission, we modify (1) to give

$$S \propto \int_A [u_i(\underline{r})]\, \sigma^{tr}_{ij}(\underline{r}) n_j(\underline{r}) d^2\underline{r} \tag{2}$$

for the signal S detected by the receiving probe. Here $\sigma^{tr}_{ij}(\underline{r})$ is the stress field that the receiver would have produced over the crack if it had been acting as a transmitter. The constant of proportionality in (2) is removed in ultrasonic practice by comparing S with the signal from a suitable calibration reflector.

Of course the dynamic COD $[\underline{u}(\underline{r})]$ is unknown - its exact determination involves solution of the elastic wave equation and boundary conditions. This brings us to our second approximation illustrated by equation (1), namely a plausible approximation to the COD. In Kirchhoff theory the COD is approximated by using simple geometrical elastodynamics, in which diffraction at the crack edge is neglected. Thus one assumes that the total field is zero on the 'dark' side of the crack, while on the 'lit' side the total field at each point is the one that would be present on an infinite tangent plane. This leads to the detected signal S being expressed as an integral over the crack face of the incident fields from the two probes, modulated by plane wave reflection coefficients.[5] In general, the double integral must be evaluated numerically.

As the authors have discussed elsewhere[2,4,5], the Kirchhoff theory can be expected to be a good approximation for crack detection near the specular direction and other cases where the crack face contribution dominates the scattering. However, inaccuracy of the Kirchhoff COD approximation near crack edges leads us to seek an alternative model of the defect-sound interaction for defects viewed in more off-specular directions. We use an elastodynamic version of Keller's Geometrical Theory of Diffraction (GTD)[7], which has been described at length in our own reports[2,4] and in the work of Achenbach and colleagues.[8] The GTD is a ray theory which includes diffraction off the crack edges as well as reflection off the crack faces. In essence each incident ray on a crack edge gives rise to cones of diffracted shear and compression rays.

Using GTD, the detected signal in ultrasonic inspection is found to bear a simple relation to the incident field and the local geometry of the crack edge at the 'glint points' or 'flash points' at which rays are diffracted from the transmitter into the receiver. In pulse-echo testing (coincident transmitter and receiver), for example, the detected signal S for a defect in the far field is given by

$$S \propto \lambda^{3/2} \left| F(\beta) \right| \left| \frac{aR}{2(a-R\cos\beta)} \right|^{\frac{1}{2}} \left| \psi^{inc} \right|^2, \tag{3}$$

where λ is the wavelength, β is the angle between the incident ray and the crack face, R is the range, a is the radius of curvature of the crack edge, and ψ^{inc} is the incident field of the probe, all measured at the glint point. Equation (3) consists of the product of four terms: (i) a factor $|\psi^{inc}|^2$ which arises from reciprocity arguments as in equation (2), (ii) an explicit frequency dependence $\lambda^{3/2}$, (iii) a geometrical term (the square root) which allows for focussing or divergence of the rays as they travel, and (iv) a <u>diffraction coefficient</u> $F(\beta)$. This last term is obtained from the exact solution[10,3] of the 'canonical' problem of the diffraction of a plane wave by a semi-infinite crack at the same angle of incidence β. The GTD also has ready application to tandem testing[3] and to the Time-of-Flight technique[11].

The GTD formula is much simpler to use in calculations than the Kirchhoff formula (2). It has the added attraction of being more satisfying physically and mathematically in that it is believed to give correctly the first term in the asymptotic expansion of the elastic wave equation, except in two types of region where GTD is known to break down. The first type of region is near the boundary of a zone where a geometrical field, such as the specular reflection from the crack face, is also present. In formula (3) this occurs when $\beta = 90^{o}$, where $F(\beta)$ becomes infinite. The second type of region is near caustics of the diffracted field, exemplified by the case $a = R \cos \beta$ in equation (3) where again the formula diverges as two or more adjacent rays intersect at the receiving probe. However, it should be noted that the Kirchhoff theory does not have any singular behaviour at either geometrical boundaries or caustics, and consequently a judicious choice of Kirchhoff theory or GTD allows us to calculate the detected signal accurately in practically all circumstances.

As discussed in the Introduction, we can validate these models by comparisons with exact solutions of the elastic wave equation and with experimental measurements on carefully chosen specimens. Figure 1 compares the Kirchhoff and GTD descriptions of the back-scattered field from a strip-like crack with the exact results, which are obtained from a high-frequency asymptotic expansion due to Keogh[12]. The GTD result agrees precisely with the first term in this expansion, and can be seen to be reasonably accurate at all angles of incidence, except for the sharp spike at the critical angle of about 33^{o}. Here the higher-order terms in Keogh's expansion are of comparable magnitude to the first term and 'cancel out' the spike to give the smooth exact result. Kirchhoff theory is reasonably accurate near normal incidence, out to about the first two sidelobes, but becomes totally misleading beyond this region; this is as expected. Similar behaviour occurs in the corresponding comparison for the penny-shaped crack[4], where the recent exact results of Martin and Wickham[13] were used. The only major difference is that the GTD breaks down near normal incidence where the probe lies on the axial caustic.

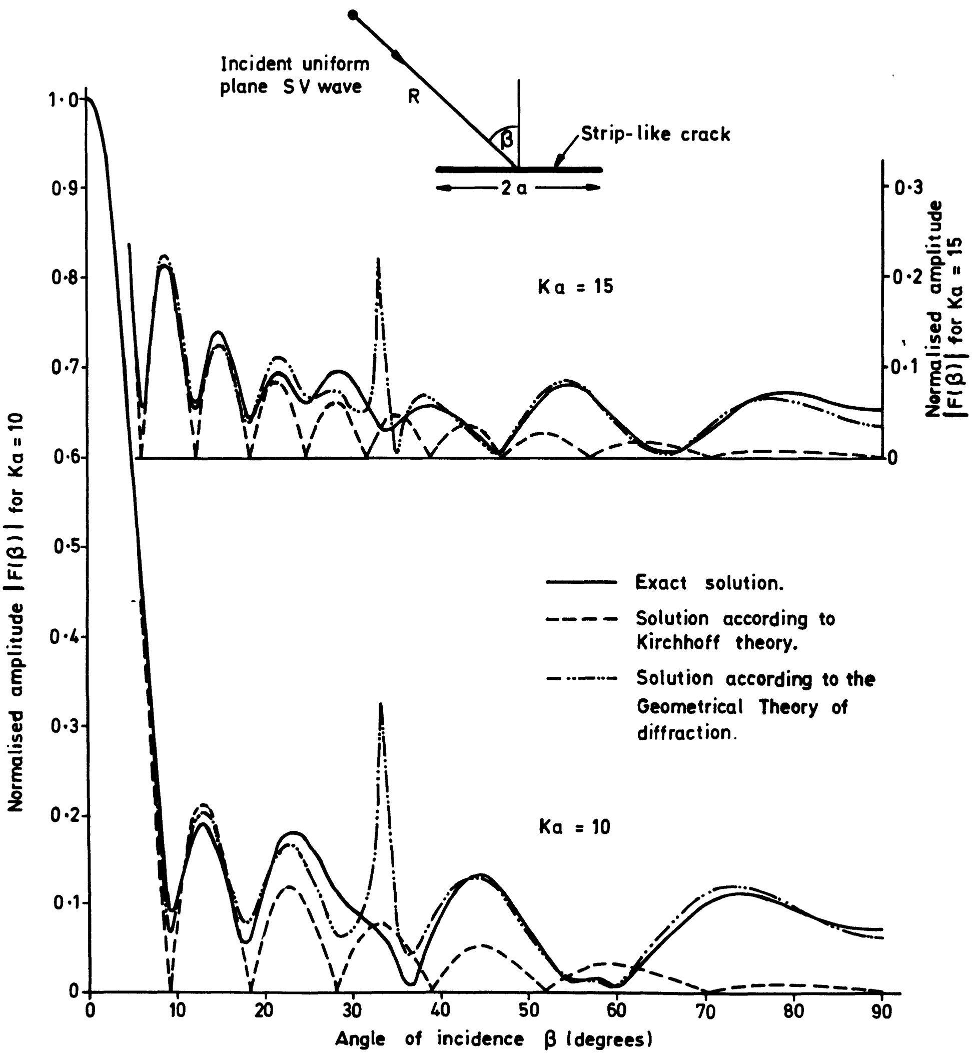

Fig. 1. Scattering of a plane SV wave in steel by a smooth strip-like crack of width 2a. The far field of the back-scattered SV waves according to Kirchhoff theory and GTD, compared with the exact solution, for Ka = 10 and Ka = 15 (K = shear wavenumber).

An example of the comparison of GTD results with experiment is shown in Figure 2. The scatterer was a straight fatigue crack grown in a 125 mm radius circular 'compact tension' specimen, with the block machined so the crack tip was at the centre. The results are for pulse-echo detection of the crack tip using a 0° compression wave probe beam directed radially from the cylindrical face of the block.

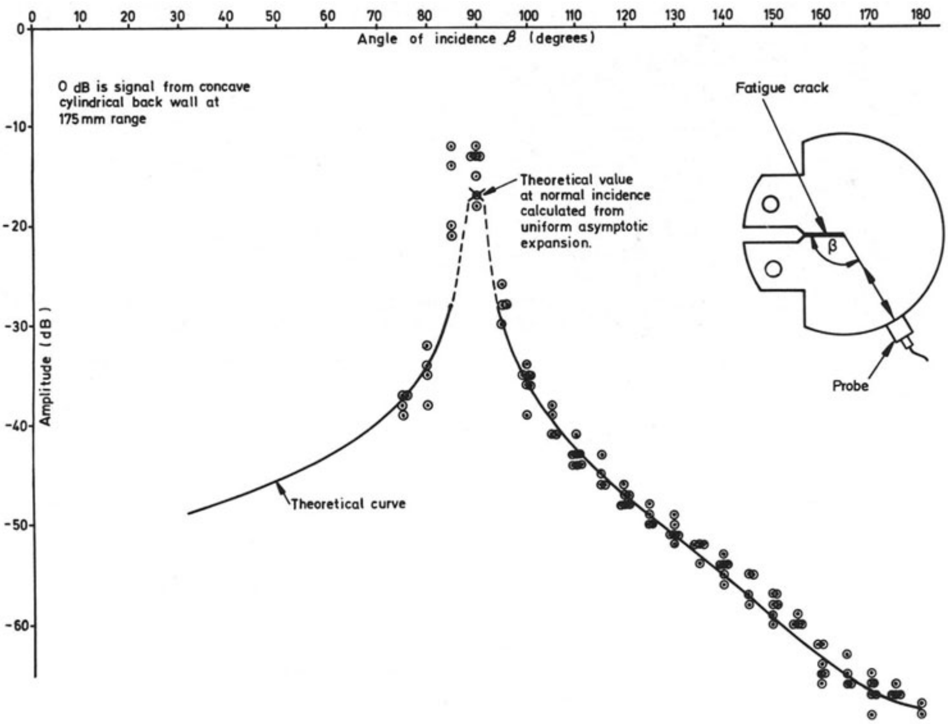

Fig. 2. Comparison of GTD with experiment for the diffraction of compression waves from a fatigue crack tip.

Model of the Probe Beam

Values for the complex incident fields in equations (2) and (3) are best provided using a model of the ultrasonic beams radiated by probes. We have derived a closed form expression for the field based on the approximation

$$2J_1(x)/x \simeq (1- x^2/15) \exp (-x^2/15) \tag{4}$$

to the far field of a piston-like circular probe. The model field is a good approximation at ranges as close as about ¾ of a nearfield length, and at lateral distances out to about the peak in the first sidelobe - that is, to about as far as the usual piston model is itself a realistic approximation to the beam of any actual probe. Figure 3 compares the model beam shape with that obtained experimentally for a ½" diameter probe.

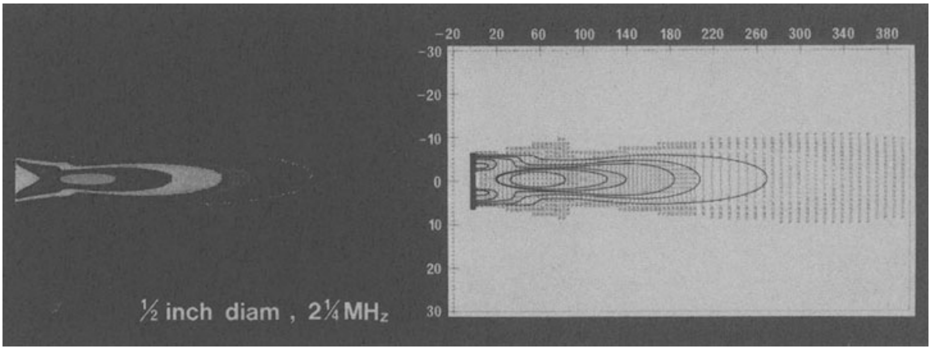

Fig. 3. Comparison between model (right) and experimentally measured (left) beam shapes for normal compression probe beam in water.

Geometry of the Inspection

Finally we mention the way in which the geometries of the component, the surfaces scanned and the scanning mechanism are incorporated into the model. Our computer programs for evaluating the scattered field relate to a coordinate system centred on the defect. However the probe beam is most naturally calculated in 'probe' coordinates, where one coordinate axis coincides with the beam axis, while the probe's motion over the component's surfaces is best expressed in a coordinate system related to the component's geometry. For the model examination of a flat block these three coordinate systems are simply related by translations which allow for probe position and by rotations which allow for tilt and skew of the probe and of the defect. In other cases the relations are more complicated. For example, in a recent application to the nozzle-to-shell weld inspection of a PWR, account had to be taken of the geometry of the nozzle region and the articulations of the scanning mechanism used in the CEGB's automated inspection of the PWR nozzle specimen, undertaken as part of the PISC II programme. Several other case studies in different geometries have demonstrated the adaptability of our overall model.

EXTENSIONS OF THE MODEL USING GENERALISED RAY THEORY

We currently hold the view that the model outlined above is quite serviceable and could be usefully applied to many further practical inspection problems without significant modification.

However it does have several limitations or inelegant aspects which we aim in time to improve upon. Among these are:-

(a) It is rather dubious inside the nearfield, particularly for high angle (e.g. 70°) probes.
(b) It requires a judicious use of the complementary Kirchhoff and GTD scattering theories. The Kirchhoff theory is physically less attractive than the GTD and usually requires much more computing time.
(c) It does not make an explicit and fully correct treatment of the time-dependence of the detected pulses. In particular, the change in transmitted pulse shape with angle from the probe beam axis is not acknowledged.
(d) It does not include propagation through an irregular coupling layer or through an anisotropic medium, such as a hand-ground austenitic steel weld.

In addition, of course, our model does not apply to rough defects. As we have discussed[2,14], treatment of roughness on the crack faces is still a major challenge to applied mathematics, notwithstanding the vast literature on acoustic Kirchhoff theory of rough-surface scatter, or on elastic perturbation theory approaches to weakly rough cracks, such as the recent work by Ramsdale[15].

Leaving roughness aside, the other problems listed above can in principle be fruitfully developed using ray theory or more sophisticated extensions of ray theory based on uniform asymptotic solutions of the elastic wave equations in canonical problems. For example, Wickham and Coffey[16] have published a model of a two-dimensional ultrasonic angled shear wave probe based on ray-theoretic ideas. The field was given by the vector sum of contributions from the geometrical field from the face of the piezo-electric crystal, and from the radiated waves from the top and bottom crystal edges. The neighbourhoods of the geometrical field boundaries were treated using uniform asymptotics, and involved tracing the refraction of these boundaries from the Perspex wedge forming the probe shoe into the steel component. This model offers a powerful and elegant way of treating the probe beam at short range, and in principle can be extended to include inter-facial coupling layers and propagation into anisotropic materials. In this context the reader may like to note the work of Wooldridge[17] at the NDT Applications Centre, who has used non-uniform ray tracing methods to give a satisfactory quantitative description of an ultrasonic beam passing through an irregular, inhomogeneous, anisotropic layer of austenitic strip cladding on a ferritic pressure vessel wall.

As a further illustration of ray-theoretic methods we have followed Lewis and Boersma[18] and Achenbach et al[8] in obtaining a modification to equation (3) valid uniformly in the region near

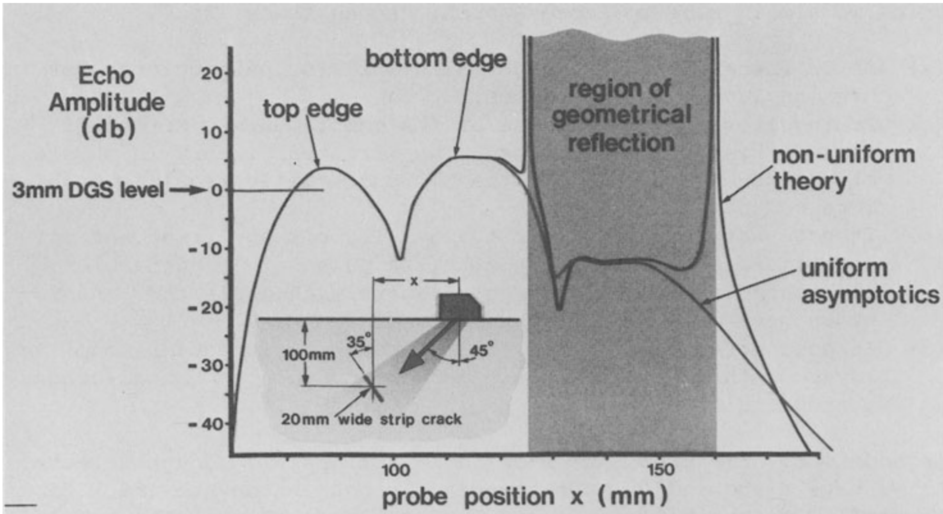

Fig. 4 Comparison of non-uniform and uniform asymptotic theories for pulse-echo probe inspecting strip-like crack. Frequency = $1\frac{1}{2}$ MHz

$\beta = \pi/2$, the boundary of the geometrically reflected field. First we note that the total non-uniform response in this region is given by vector addition of the edge wave term (3) and a geometrically reflected term $H(\pi/2-\beta)\, S_{geom}$, where the Heaviside function H describes the discontinuity from 'lit' to 'dark' sides of the boundary. It can be shown that the uniform response near the reflection boundary is given by replacing this Heaviside function by the function

$$G(x) = \tfrac{1}{2}\,\mathrm{erfc}\,(-e^{-i\pi/4}x) + \tfrac{1}{2}\,\pi^{-\frac{1}{2}}\,e^{i\pi/4}\,x^{-1}e^{ix^2}, \qquad (5)$$

where $x = 2\sqrt{KR}\,\sin\tfrac{1}{2}(\pi/2-\beta)$. The effect of this is illustrated in Figure 4, in which the ultrasonic responses according to non-uniform and uniform theory are compared when a pulse-echo probe inspects a strip-like crack tilted at 10° to the probe beam direction. The uniform response remains finite and continuous at the reflection boundaries of the top and bottom edges.

Several authors have written about the uniform asymptotics of fields near caustics, for example Ludwig[19] and Wolfe[20]. However Achenbach et al[8] (final chapter) remark upon the geometrical difficulties involved in introducing uniform corrections at caustics. They propose instead that the Elastic Green's Theorem (1) be used, but with an improved approximation to the COD above that of Kirchhoff theory. As well as the geometrical elastodynamics field, additional terms are included in the COD to allow for perturbations on the crack faces caused by diffraction at the edges due, for example, to diffracted Rayleigh waves. These extra terms are computed by combining GTD with the solutions for the COD of the canonical problems of diffraction by a semi-infinite crack. This approach appears to give good results[8]. However, the analysis is very complicated, and great care must be taken in computing the COD to avoid small errors being magnified through the Elastic Green's Theorem to give large errors in the scattered field - indeed there is no rigorous way of ensuring that this cannot happen. Despite the interest in Achenbach's approach, we still see distinct attractions in pursuing a direct approach like that of Ludwig and Wolfe, particularly in view of recent advances in the understanding of caustics in general following the application of Catastrophe Theory to their morphology and structural stability, for example by Berry and his colleagues[21].

This paper is published by kind permission of the Director-General, CEGB, North Western Region.

REFERENCES

1. J.M. Coffey, The reliability of ultrasonic inspection : developments in quantitative NDE in the United Kingdom, these Proceedings.
2. J.M. Coffey and R.K. Chapman, Application of elastic scattering theory for smooth flat cracks to the quantitative prediction of ultrasonic defect detection and sizing, J.Brit. Nucl. Energy Soc. 22 : 319 (1983).
3. R.K. Chapman, Ultrasonic reflection from smooth flat cracks: exact solution for the semi-infinite crack, CEGB Report No. NW/SSD/RR/145/81 (1981).
4. R.K. Chapman and J.M. Coffey, Ultrasonic scattering from smooth flat cracks: edge wave signals from finite cracks with curved edges, CEGB Report No. NWR/SSD/82/0034/R (1982).
5. R.K. Chapman, Ultrasonic scattering from smooth flat cracks: summary note on an elastodynamic Kirchhoff theory, CEGB Report No. NWR/SSD/82/0091/N (1982).
6. R.B. Thompson, D.O. Thompson, H.M. Burte and D.E. Chimenti, The use of field-flaw interaction theories in NDE modelling and improving reliability, these Proceedings.
7. J.B. Keller, Diffraction by an aperture, J. Appl. Phys. 28: 426 (1957).
8. J.D. Achenbach, A.K. Gautesen and H. McMaken, 'Ray methods for waves in elastic solids', Pitman, London (1982).
9. G.S. Kino, The application of reciprocity theory to scattering of acoustic waves by flaws, J. Appl. Phys. 49: 3190 (1978).
10. A.W. Maue, Die Beugung elasticher Wellen an der Halbebene, Z. Angew. Math. Mech. 33: 1 (1953).
11. J.A.G. Temple, Time of flight inspection : theory, J. Brit. Nucl. Energy Soc. 22: 335 (1983).

12. P.S. Keogh, Some exact solutions of scattering of elastic waves for plane finite cracks, Ph.D. thesis, University of Manchester, England (1983).
13. P.A. Martin and G.R. Wickham, diffraction of elastic waves by a penny-shaped crack: analytical and numerical results, Phil. Trans. Roy. Soc. Lond., to be published, November 1983.
14. J.M. Coffey, R.K. Chapman and D.J. Hanstock, The ultrasonic detectability of a postulated 'worst case' flaw in a PWR vessel, CEGB Report No. NWR/SSD/82/0045/R (1982).
15. S.A. Ramsdale, Scattering of elastic waves by a rough semi-infinite crack, Ph.D. thesis, University of Manchester, England (1983).
16. G.R. Wickham and J.M. Coffey, An improved theory of ultrasonic shear wave probes, Conference on the Evaluation and Calibration of Ultrasonic Transducers, Institute of Physics, London, 11-12 May 1977, IPC Science and Technology Press, Guildford, England (1978).
17. A.B. Wooldridge, A ray-tracing model for predicting the effects of austenitic stainless-steel cladding on the propagation of ultrasonic shear waves, CEGB Report No. NWR/SSD/82/0005/R (1982).
18. R.M. Lewis and J. Boersma, Uniform asymptotic theory of edge diffraction, J. Mathl. Phys. 10: 2291 (1969).
19. D. Ludwig, Uniform asymptotic expansions at a caustic, Comm. Pure Appl. Math. 19: 215 (1966).
20. P. Wolfe, A new approach to edge diffraction, SIAM J. Appl. Math. 15: 1434 (1967).
21. M.V. Berry, Singularities in waves and rays, Course 7 in course on Physics of Defects, ed. R. Bahan et al., Les Houches, Session XXXV, North-Holland, Amsterdam (1981).